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Bending of laminated glass

Synopsis

A new theory representing the bending of beams laminated from stiff and soft layers is presented. It is shown to be simple in concept and easily applicable to multiple laminations, and reproduces published test results with useful accuracy.

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Introduction

Glass sheets are frequently laminated together with the transparent polymer polyvinylbutyral (PVB) to build up a stiff and strong enough thickness to carry structural loads. The PVB also adds toughness and resilience to the strong but brittle glass. This composite material is commonly used in impact-resistant windows and doors and special architectural features, including load-bearing walls, floors and stairs.

Previous theories of the bending of laminations of glass and PVB have been thoroughly reviewed by Norville et al (1998) and Behr et al (1993), culminating in the theory presented by Norville et al in 1998. The present theory is suggested as an improvement in that:

1. It is simpler in concept
2. It takes into account more of the properties of the soft layer
3. It is easily applicable to multiple laminations beyond just two stiff layers

Basis of the theory

In the following description glass and PVB are assumed to be the stiff and soft materials. The complete depth of the laminated beam is initially considered to be uniformly made of glass, and then the layers actually composed of PVB are assumed to soften until they have a proportion p of the stiffness of the glass. The factor p can vary from close to 1, at low temperatures, to less than 0,3 at temperatures above 50 °C. Rate of loading will also affect p.

If it is assumed that p can vary from 1 to zero, then the bending stress distributions shown in Fig 1 will result. In Fig 1(b) the bending moment has been reduced from that applied in Fig 1(a) to keep the maximum stress constant at σ.

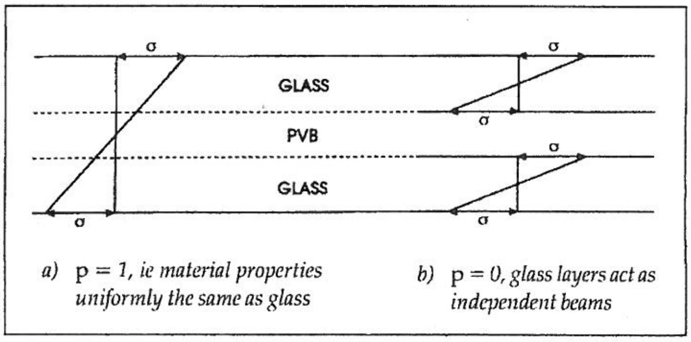


Fig 1: Bending stress distributions in a two-glass layer lamination for extremes of stiffness of the central layer of PVB

As the PVB softens and p reduces from 1 to 0, the transition of the stress distribution from that in Fig 1(a) to that in Fig 1(b) is assumed to occur in linear proportion to the factor p. This is illustrated in Fig 2.

Moments carried by laminated beams

When the PVB has proportion p of the stiffness properties of the glass, the bending moment to bring the maximum stress in the glass to σ can be expressed in terms of p and the layer thicknesses.

Consider as an example a beam made up of two glass layers of thickness s with a thickness t of PVB between them, as shown in Fig 2. The moment carried by the PVB layer is given by:

$$M_{AB} = p\sigma_{B0}Z_{AB} \qquad (1)$$

where $\sigma_{B0} = \sigma\left(\frac{t}{t+2s}\right)$ from the stress diagram when $p = 1$

Z_{AB} = section modulus for unit width = $t^2/6$

The moment carried by the glass layers BC is derived as follows:

$$\sigma_{BC1} = \sigma_{B0} - \delta\sigma_{BC} \qquad (2)$$

where $\delta\sigma_{BC} = (1-p)(\sigma_{B0} + \sigma)$

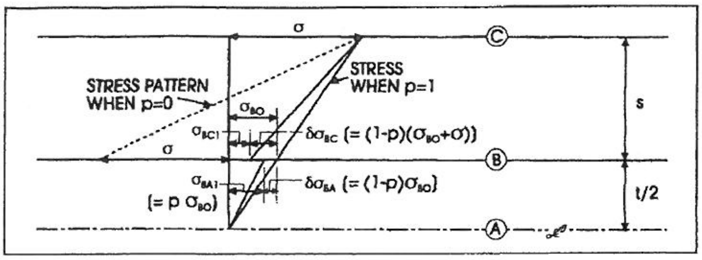


Fig 2: Intermediate stage 1 in the transformation of stress patterns as p reduces from 1 to 0

The stress pattern of σ and σ_{BC1} can be expressed as the superposition of a uniform stress σ_{uBC} and a bending stress σ_{bBC}, where:

$$\sigma_{uBC} = (\sigma + \sigma_{BC1})/2 \quad (3)$$

and

$$\sigma_{bBC} = (\sigma - \sigma_{BC1})/2 \quad (4)$$

(Note that the value of σ_{BC1} substituted into these two equations must be negative for lower values of p.)

The bending moment about the centreline of the laminated beam contributed by the two layers BC, per unit width, is then given by:

$$M_{BC} = 2[\sigma_{uBC}\, s(s/2 + t/2) + \sigma_{bBC}\, s^2/6] \quad (5)$$

The total bending moment carried is then:

$$M_{ABC} = M_{AB} + M_{BC} \quad (6)$$

Deflections of laminated beams

The deflection of the whole beam can be taken as the deflection, due to bending only, of one layer of it. Again consider as an example a beam with two glass layers.

The bending moment carried by one glass layer, about the centreline of the glass layer, is given by:

$$M_{1BC} = \sigma_{bBC}\, s^2/6 \quad (7)$$

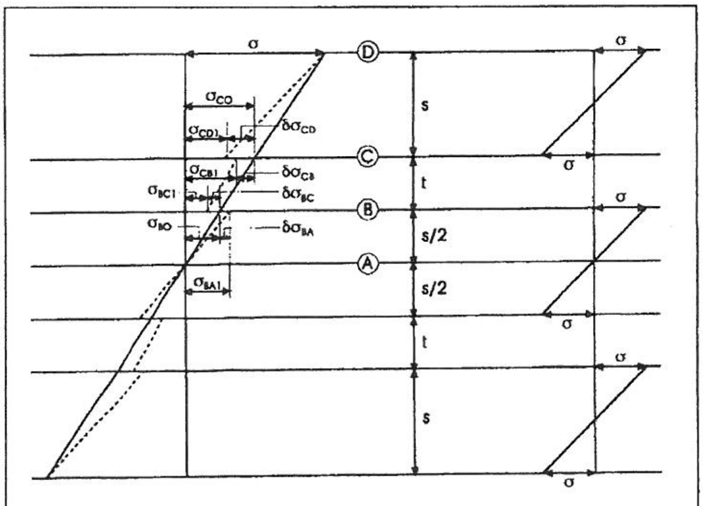
where σ_{bBC} is given by Eqn 4.

Bending deflection is then given by integration of the differential equation:

$$\frac{d^2y}{dx^2} = \frac{M_1(x)}{EI_1} \quad (8)$$

where:

$M_1(x)$ = M_1 at distance x from the deflection reference point
E = Young's modulus for glass
I_1 = second moment of area of unit width of a glass layer $= s^3/12$



a) Stresses when soft layers have proportion p *of the stiffness of the stiff layers*　　*b) Stresses when* p = 0

Fig 3: Laminated beam with three stiff layers and two soft layers

Beams laminated with multiple layers

The derivation of an expression for the bending moment carried, in terms of the extreme fibre stress s and the proportion p of the stiff layers' properties possessed by the soft layer, follows the same method as already described for a beam with two stiff layers.

The expression for a beam with three stiff layers will be derived here to show that no new difficulties arise (Fig 3).

Layers AB

$$\sigma_{BA1} = \sigma_{B0} + \delta\sigma_{BA} \quad (9)$$

$$\text{where } \sigma_{B0} = \left(\frac{s/2}{t + 3s/2}\right)\sigma = \left(\frac{s}{2t + 3s}\right)\sigma \quad (10)$$

$$\text{and } \delta\sigma_{BA} = (1 - p)(\sigma - \sigma_{B0}) \quad (11)$$

$$\text{Then } M_{AB} = \sigma_{BA1}\, s^2/6 \quad (12)$$

Layers BC

$$\sigma_{BC1} = \sigma_{B0} - \delta\sigma_{BC} \quad (13)$$

where σ_{B0} is given by Eqn 10

$$\text{and } \delta\sigma_{BC} = (1 - p)\sigma_{B0} \quad (14)$$

$$\text{Also } \sigma_{CB1} = \sigma_{C0} - \delta\sigma_{CB} \quad (15)$$

$$\text{where } \sigma_{C0} = \left(\frac{t + s/2}{t + 3s/2}\right)\sigma = \left(\frac{2t + s}{2t + 3s}\right)\sigma \quad (16)$$

$$\text{and } \delta\sigma_{CB} = (1 - p)\sigma_{C0} \quad (17)$$

The stress pattern of σ_{BC1} and σ_{CB1} can be expressed as the superposition of a uniform stress σ_{uBC} and a bending stress σ_{bBC}, where:

$$\sigma_{uBC} = (\sigma_{BC1} + \sigma_{CB1})/2 \quad (18)$$

$$\text{and } \sigma_{bBC} = (\sigma_{BC1} - \sigma_{CB1})/2 \quad (19)$$

The bending moment about the centreline of the laminated beam contributed by layers BC, per unit width, is then:

$$M_{BC} = 2[\sigma_{uBC}\, t(t/2 + s/2) + \sigma_{bBC}\, t^2/6] \quad (20)$$

Layers CD

$$\sigma_{CD1} = \sigma_{C0} - \delta\sigma_{CD} \quad (21)$$

$$\text{where } \delta\sigma_{CD} = (1 - p)(\sigma_{C0} + \sigma) \quad (22)$$

$$\text{Then } \sigma_{uCD} = (\sigma + \sigma_{CD1})/2 \quad (23)$$

$$\text{and } \sigma_{bCD} = (\sigma - \sigma_{CD1})/2 \quad (24)$$

$$\text{giving } M_{CD} = 2[\sigma_{uCD}\, s(t + s) + \sigma_{bCD}\, t^2/6] \quad (25)$$

The total bending moment $M_{ABCD} = M_{AB} + M_{BC} + M_{CD}$.

Deflections of beams with multiple layers

The deflection of one of the layers, owing to the bending moment it carries about its own mid-depth, can be taken as the deflection of the whole beam.

Using an outer glass layer, the relevant bending moment is given by:

$$M_{1CD} = \sigma_{bCD} s^2/6$$

where σ_{bCD} is given by Eqn 24.

Integration of Eqn 8 is used to determine values of deflection.

Comparison with test results

Behr et al (1993) reported tests on laminated beams with two layers of glass and a layer of PVB. Measurements were made of centre-span deflection and strain (interpreted by the authors into stress) at the extremities of the beam, at three different temperatures. They also report a bending test of a monolithic piece of glass (presumably at a temperature around room temperature), which can be interpreted to give a value of Young's modulus for the glass.

Using the theory described in this paper, values of p needed to produce the measured stress values are as follows:

Temperature °C	p
0	0,94
23	0,75
49	0,28

The beam deflections were then predicted with the theory, using these values of p and the single value of Young's modulus available. Comparison with the measured values is shown in Fig 4.

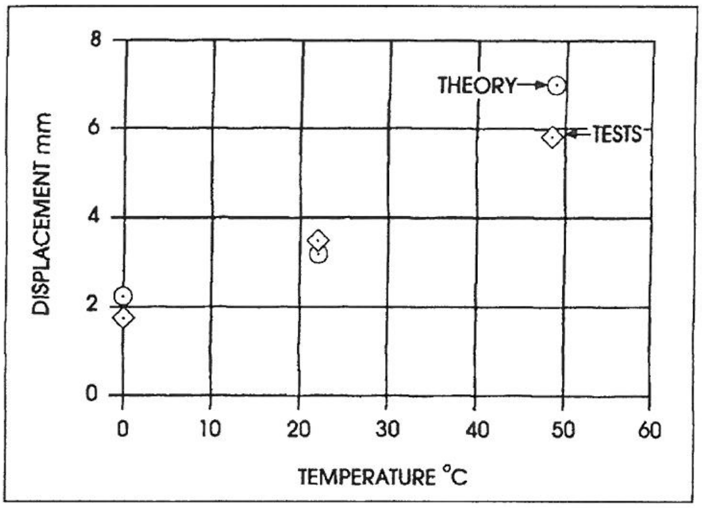


Fig 4: Comparison of theoretical predictions of deflection to measured values

Conclusions

1. The theory presented is conceptually simple and complies with the upper and lower bounds of the properties of the PVB, ie when it has properties that are equal to glass or negligibly small.
2. The theory enables stresses and deflections to be predicted, and can be easily extended to any number of laminations.
3. The theory shows correlation with published measured values that is good compared with the variability of glass strength and as accurate as theory recently proposed by Norville (1998).

References

1. Behr, R A, Minor, J E and Norville, H S. 1993. Structural behaviour of architectural laminated glass. *J of Structural Engineering, ASCE*, 119(1): 202-222.
2. Norville, H S, King, K W and Swofford, J L. 1998. Behaviour and strength of laminated glass. *J of Engineering Mechanics, ASCE*, 124(1): 46-53

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Written discussion on the technical papers in this issue of the *Journal* will be accepted until 31 July 1998. This, together with the authors' replies, will be published in the Fourth Quarter 1998 issue of the *Journal*, or the issue thereafter. For the convenience of overseas contributors only, the closing date for discussion will be extended to 31 August 1998. Discussion must be sent to the Directorate of SAICE.

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