1991

TECHNICAL NOTE

Pressuremeter interpretation by progressive relaxation

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Introduction

A need exists in geotechnical engineering for a model of soil stress-strain behaviour that is simple in concept and that has easily obtainable parameters. Because of this need, the linear isotropic elastic model, with just two constants, is widely used in practice. A significant improvement in performance over that model, while remaining simple in concept, is offered by one that allows for variable moduli.

This paper presents an easily applied method of obtaining the parameters for a model in which the bulk and shear moduli of the soil depend upon the current stress-strain states.

Method

The soil around the pressuremeter is assumed to be divided into annular rings of uniform thickness, the interfaces between which are numbered as shown in Fig 1. (The thickness and number of these rings are discussed later.) Initially, only one ring of soil is considered to exist round the pressuremeter and the first stage of radial stress σ_r is applied to the inside of the ring at radius r_1. The movement at the outside of the soil ring is prescribed to be zero and, using Lamé's[1] theory, the radial expansion of the pressurementer (δr at radius 1) is calculated. The choice of the elasticity parameters used is described below under 'Material model'.

The calculation of δr at r_1, using Lamé's theory, is carried out in two stages. First the induced radial stress at r_2 (that is $\sigma_{r,\,r_2}$) is found for the applied radial stress at r_1 and with the condition that $\delta r_{r_2} = 0$. Second, for the inner and outer radial stresses, $\sigma_{r,\,r_1}$ and $\sigma_{r,\,r_2}$, the radial movement at the pressuremeter, δr_{r_1}, is calculated. (Note: the algebraic derivations of these relationships are not complex, but are too voluminous to be included in this technical note. If interested readers contact the author, a copy will be provided.)

A second annular ring is then added and calculations as previously described are applied to both rings, starting with the outside one, in the following manner (use Fig 1 as a guide when reading the next paragraph).

The radial stress at r_2 (calculated when δr_{r_2} was assumed to be zero) is now the applied radial stress on the ring between r_2 and r_3, and the induced radial stress at r_3 is calculated for δr_{r_3} prescribed to be 0. For $\sigma_{r,\,r_2}$ and $\sigma_{r,\,r_3}$, δr_{r_2} is then found. This value of δr_{r_2} is now the prescribed movement at r_2, used to calculate a new value of $\sigma_{r,\,r_2}$ induced by $\sigma_{r,\,r_1}$, the increment of stress applied by the pressuremeter.

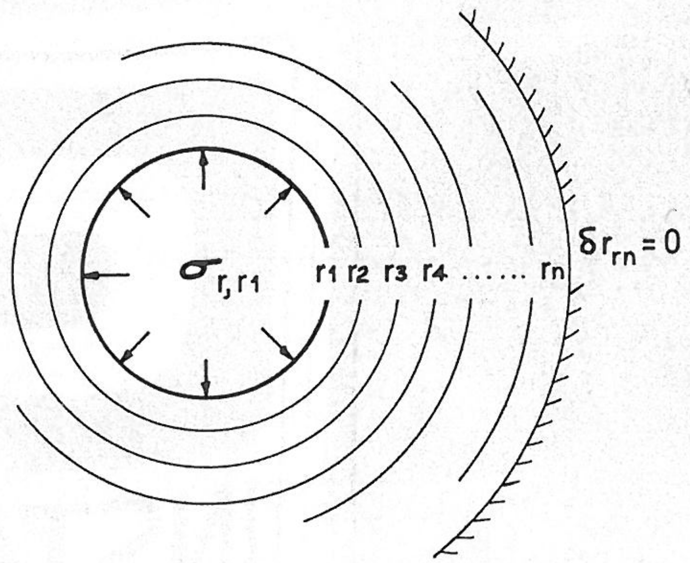
Finally, δr_{r_1} is calculated using $\sigma_{r,\,r_1}$ and the most recently calculated value of $\sigma_{r,\,r_2}$.

In the procedure described for two rings, it can be seen that δr_{r_2} is now free to move, whereas with just one ring, r_2 was at the outside of the soil annulus around the pressuremeter and therefore δr_{r_2} was prescribed to be zero. The relaxation of δr_{r_2} reduces $\sigma_{r,\,r_2}$ and hence allows δr_{r_1} to increase. This progressive relaxation of the body of soil round the pressuremeter continues with the inclusion of further annual rings at the outside. Fig 2 shows the effect on the calculated expansion of the pressuremeter of increasing the number of annular rings of thickness (pressuremeter diameter/8).

The addition of further rings of soil can be stopped when the effect on δr_{r_1} becomes negligible. In the case of the example shown in Fig 2, this was at a radius of 2,5 times the pressuremeter diameter. Hence the number of annular rings of soil is chosen automatically. Because each ring has its own values of elastic moduli, the thickness of the rings will influence the calculated value of δr_{r_1}. However, this influence is not great and a thickness of pressuremeter diameter/8 is a good compromise between accuracy and speed of calculations.

Material model

The material model considers separately the behaviour of the soil under the hydrostatic and shear components of the stress state acting on it.



$r_1, r_2..r_n$ = Identification numbers of the interfaces between annular rings
σ_r, r_1 = Total radial stress at interface r_1, ie at the pressuremeter
$\delta_{r_{r_n}}$ = Radial movement of interface number rn

Fig 1: Division of soil around pressuremeter into annular rings

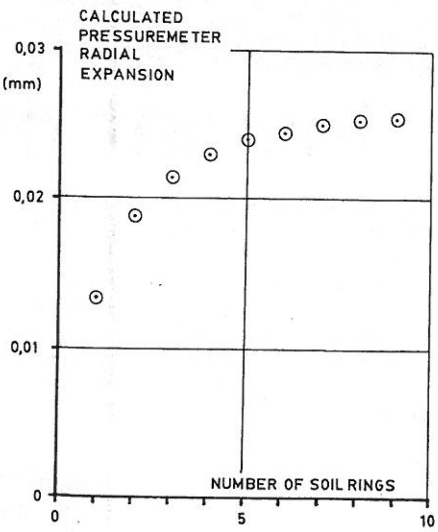


Fig 2: Effect of the number of rings of soil around the pressuremeter on its calculated radial expansion under one increment of stress

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Soil behaviour under hydrostatic stress

For the expansion of a cylinder in an isotropic, linear elastic material, there is no net change in the mean direct stress (ie the hydrostatic component of the stress state). However, because the soil behaviour is not linear, there will be a net increase in compressive stress on the soil around the pressuremeter. It is assumed that the behaviour under hydrostatic stress is that given by the results of a laboratory test on an undisturbed sample of the soil. Domaschuk and Wade[2] suggested that the following function can be used to represent that behaviour:

$$\sigma_m = \alpha \varepsilon_v^{\beta} \qquad (1)$$

where

σ_m = hydrostatic stress
= $(\sigma_1+\sigma_2+\sigma_3)/3$
ε_v = volumetric strain
α and β = constants found by plotting the test results on axes of log σ_m against log ε_v

A typical test result and the Eqn 1 functions to represent it are shown in Fig 3.

In the calculation of radial stresses and movements that pertain at the end of a stage of load application by the pressuremeter, the secant bulk modulus, K_{sec}, is needed. An expression for K_{sec} is derived as follows:

$$K_{sec} = \frac{\sigma_m}{\varepsilon_v}$$

But from Eqn 1 $\varepsilon_v = \left(\frac{\sigma_m}{\alpha}\right)^{1/\beta}$

Substituting for ε_v gives $K_{sec} = \alpha^{1/\beta}\,\sigma_m^{(1-1/\beta)}$ (2)

Soil behaviour under shear stress

It has been found by Kondner and Zelasko[3] and Duncan and Chang[4] that a hyperbolic relationship successfully models shear behaviour. This relationship is of the form:

$$\tau = \frac{\gamma}{\frac{1}{G_{init}} + \frac{\gamma}{\tau_{asymp}}} \qquad (3)$$

where

τ and γ = the corresponding shear stress and strain on any plane
G_{init} = initial s. ,ar modulus at very low strains
τ_{asymp} = asymptotic stress limit of shear stress τ that the hyperbolic Eqn 3 approaches.

These parameters are illustrated in Fig 4. The value of the secant shear modulus is then given by:

$$G_{sec} = \frac{\tau}{\gamma} = \frac{1}{\frac{1}{G_{init}} + \frac{\gamma}{\tau_{asymp}}} \qquad (4)$$

It is convenient, for simplicity of expressions, to use the plane on which maximum shear stress and strain occur (assumed coincident) to calculate the value of γ for use in Eqn 4.

The two parameters G_{init}, which is a measure of shear stiffness, and τ_{asymp}, which is a measure of strength, are required to be found. Both could be found from the pressuremeter test, but because different drainage conditions will exist in the actual design situation, where movements need to be predicted, the shear strength parameters used then in the soil model will need to be appropriate for that situation. For this reason it is proposed that c and ϕ from laboratory shear tests with drainage similar to the pressuremeter test be used to interpret it. The same type of laboratory test would then be used to give c and ϕ for the drainage conditions of the design situation.

The relationship between τ_{asymp} (ie maximum shear stress on Mohr's circle) and c, ϕ is illustrated in Fig 4 and is given by:

$$\tau_{asymp} = c\cos\phi + \frac{(\sigma_r + \sigma_\theta)}{2}\sin\phi \qquad (5)$$

This leaves G_{init} as the only unknown parameter from the soil model to be found from interpretation of the pressuremeter test. Its value is found by varying it in a process of trial and error, until the pressuremeter results graph of radial stress against radial expansion has been reduced by the $\sigma_{r_{r_1}}$, against $\delta_{r_{r_1}}$ graph from the calculations described above.

Example of application

Fig 5 is the graph of probe pressure (corrected for membrane stiffness) against radial expansion of a 'Camkometer' self-boring pres-

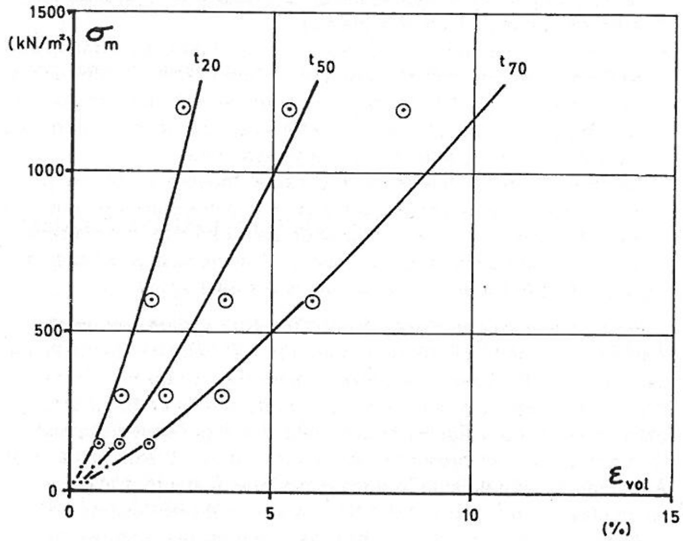

Fig 3: Results of an isotropic consolidation test on a sample of Gault clay, for various percentages of complete consolidation

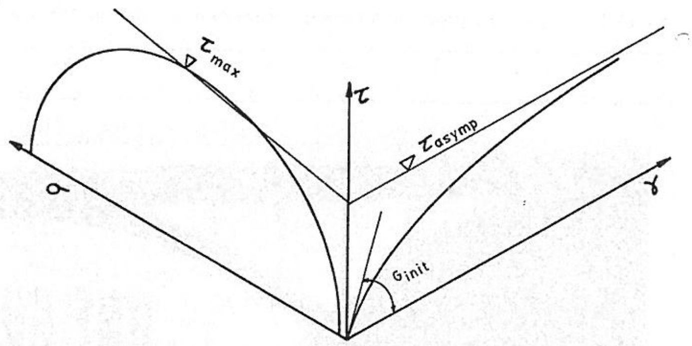

Fig 4: Graphical representation of the soil behaviour in shear assumed in the model

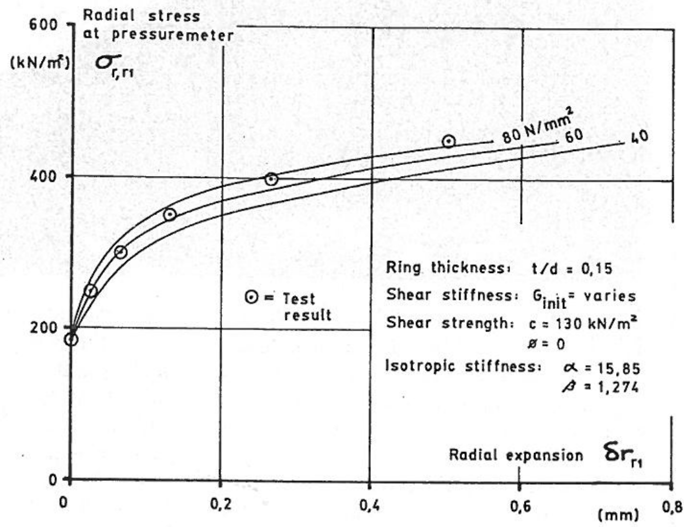

Fig 5: Pressuremeter test result in Gault clay, compared with calculated simulation for varying G_{init}

suremeter in a stiff clay (the Gault clay in eastern England). To interpret it, it is first necessary to choose the appropriate degree of consolidation curve from the isotropic consolidation test results shown in Fig 3, which are from a test done on a sample from the same site and depth. The assessment of the degree of consolidation taking place during a pressuremeter test is very difficult. A simplified approach has been adopted as follows:

1. Assume that an instantaneous rise in radial stress at the pressuremeter of 100 units is carried entirely by a rise in pore pressure and the radial distribution of the pore pressure is as given by linear elasticity theory (ie Lamé's theory).
2. Using the finite difference solution for radial flow consolidation and assuming free drainage boundaries at the pressuremeter and at four diameters away from the pressuremeter, calculate the dissipation in pore pressure at radial intervals between those boundaries over the period of time that the test was done.
3. Because more of the measured radial movement of the pressuremeter is caused by the consolidation of the soil close to it than of that further away, a weighted average is found for the degrees of consolidation calculated in stage 2. The method of weighting is according to the elastic distribution of radial stress.

From such a weighted assessment, the appropriate degree of consolidation for the test in this example is 20 per cent, hence the 20 per cent points on Fig 3 were used to get α and β for the bulk modulus function, Eqn 2. Next it is necessary to choose values of shear strength parameters c and ϕ that are appropriate to a 20 per cent dissipation of the change in pore pressure that is induced in a triaxial shear test. Again this is very difficult, but from available test information on the Gault clay, values of c = 130 kN/m² and ϕ = 0 were estimated.

The calculated simulation of the pressuremeter test is shown on Fig 5 for three values of G_{init}. In a practical situation where soil movements are to be predicted, the majority of the soil will be well below failure, say in a range of stress up to about 350 kN/m² on Fig 5. For this range, G_{init} = 60 MN/m² gives as good a fit as would be needed in practice. The soil model would then be used (probably with the finite element method) with this value of G_{init} and the values of α, β, c and ϕ appropriate to the drainage conditions that will exist in the design situation where movements are to be predicted.

For the five increments of stress shown in Fig 5, a calculation of the corresponding radial expansions took just two minutes with compiled BASIC on an IBM PC. With this degree of speed and operational convenience, it is quite easy to examine the sensitivity of the pressuremeter test to variations in the other parameters in the soil model as well as G_{init}.

Conclusions

1. The progressive relaxation method of pressuremeter interpretation can be applied with a variable moduli model of soil behaviour.
2. By the inclusion of laboratory test results in the soil model and by trial and error choice of the initial shear stiffness modulus, the pressuremeter test result can be closely reproduced.
3. The sensitivity of the reproduction of the pressuremeter test result to variation in the parameters that define the soil's behaviour can be calculated easily and quickly.
4. Having found the parameters for the soil model that enable the pressuremeter test to be reproduced, parameters for the drainage conditions that apply to the design situation can then be substituted in the model.

References

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2. Domaschuk, L, and Wade, N H. A study of bulk and shear moduli of a sand. *Proc*, ASCE, J Soil Mechs and Fdn Div, SM2, March 1969, pp 561-581.
3. Kondner, R L, and Zelasko, J S. A hyperbolic stress-strain formulation for sands. *Proc*, Panamerican Conference on Soil Mechs and Fdn Eng, 1963, Paper Ib/12, pp 289-324.
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