THE ADOPTION OF CP2012 TO DESIGN VIBRATING FOUNDATIONS WITH PILES

Mitchell Gohnert, Irvin Luker
University of the Witwatersrand, School of Civil and Environmental Engineering, Johannesburg, South Africa
Colin Morris
Fluor Daniel, South Africa

ABSTRACT

CP 2012 is used extensively throughout the world to design machine foundations for vibrations. The code, however, does not give practical advice on how to design a foundation with piles. CP 2012 models the soil as a system of undamped individual springs. A pile group may similarly be modelled as a system of springs, determined from the geometric and material characteristics of the piles. The stiffness of a pile group is expressed in the same form as sub-grade reactions, permitting the use of the same dynamic equations given in CP 2012; thus, the code may be used for both cases—soil and piles The derivation of the equations are given and compared to a simple finite element model.

KEYWORDS

Piling, foundations, dynamics, vibrations, design.

1 INTRODUCTION

Designing foundations with piles for vibrating machinery is a difficult task for the simple reason that practical design methods are not readily available or published in codes of practice [1, 2]. A dearth of information on the subject has caused some designers to resort to commercial computer programmes, such as PIGLET [3] and REPUTE [4], which are capable of analyzing pile groups.

CP 2012, the Code of Practice for Foundations for Machinery [2], is widely used in many parts of the world. The code provides explicit instructions on how to determine natural frequencies and amplitudes of vibration based on the assumption that the foundation can be modelled as a system of undamped individual springs [2, 5, 6].

The foundation block is assumed rigid and the spring stiffnesses are derived from the properties of the soil. The code makes reference to Barkan's [7] method to determine the soil stiffnesses (referred to as subgrade reactions). Since the majority of foundations supporting vibrating machinery are founded on soil, the code is applicable to most cases. However, when designing a foundation with piles, the method of design is not explicitly given by the code. Therefore, the objective is to present a simple method to determine the stiffnesses of a pile group which may be used in conjunction with CP 2012. The proposed method is similar to the design of a foundation block on soil. As mentioned, CP 2012 models the soil as a system of undamped individual springs. A pile group is similarly modelled as a system of springs [1, 8].

2 PILING ASSUMPTIONS

Sources of resistance to piling forces are shaft friction and end bearing. Vibrations, however, may diminish the frictional resistance along the shaft, particularly in coarse soils. Furthermore, the imposition of lateral forces on the shaft may compact the surrounding soil, resulting in further diminishing the frictional resistance. For these reasons, the frictional resistance along the shaft should be ignored in piles subjected to vibrations. Only where shear and moment fixities of the toe of the pile are considered is stress from the soil on the shaft may be taken into account. In general, piles applied to vibrating foundations should be "end bearing type" and the shaft assumed to be free. Longitudinal settlement of the toe of the pile should be negligibly small. To achieve this, the pile should preferably be founded on rock or sufficiently far into dense or stiff stratum so that an "effective length" of the pile may be estimated. This is the equivalent free length of a pile on a rigid base, such that its longitudinal stiffness is the same as that of the actual pile. Shear and moment fixities of the toe of the pile are achieved by sufficient embedment of the toe into a stiff and strong stratum. If a pinned toe connection is assumed, only shear fixity is required. Although forces developed from shear across the base is usually small, sufficient shear resistance can be developed from penetration of the pile by 1.5 diameters into a stratum classified as "rock". Moment fixity resistance should only be assumed if powerful drilling equipment has enabled sufficient penetration into a stiff and strong stratum. Specialist geotechnical investigation and assessment of such a situation will be needed.

3 PILE GROUP STIFFNESS EXPRESSED AS A SUBGRADE REACTION

The soil stiffness is referred to as the subgrade reaction. The value of stiffness, in units of stress per unit deflection, is called the coefficient of subgrade reaction. Four coefficients are used—the coefficient of uniform compression (c_u), the coefficient of non-uniform compression (C_ϕ), the coefficient of uniform shear (c_r) and the coefficient of non-uniform shear (c_s). These coefficients are defined in terms of the pile group stiffnesses, but expressed in the same form as coefficients of subgrade reaction, as described in the following sections 5.1 to 5.4.

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3.1 Coefficient of uniform compression

The coefficient of uniform compression is determined from the spring system illustrated in Fig. 1. The foundation block is assumed rigid and the springs schematically represent the elastic properties of the piles.

As a vertical load (R) is applied to the foundation block, a uniform deformation of Δ will occur in the piles. A pile configuration, where the centre of stiffness coincides with the centre of mass of the machine and foundation, is a necessary prerequisite to ensure a uniform deformation. The vertical stiffness of the piles (k_v), is simply the load divided by the deformation.

$$k_v = R / \Delta \tag{1}$$

Since piles may be vertical or raked, the calculation of stiffness must consider both cases, as illustrated in Fig. 2. As depicted, the total deformation of the pile may be broken down into two fundamental deformations—axial and shear. The axial deformation is represented by the symbol Δ_a and the shear deformation (side sway) is represented by Δ_s. Each of these deformations is associated with reactions R_a (axial) and R_s (shear). The stiffness of the pile depends on the moment fixity of the bearing end.

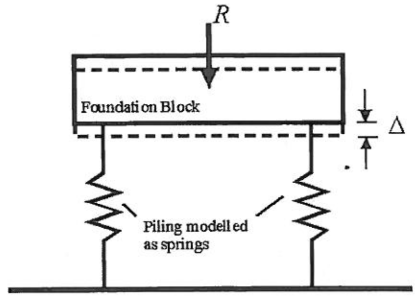

Fig. 1: Foundation spring system for vertical deformations

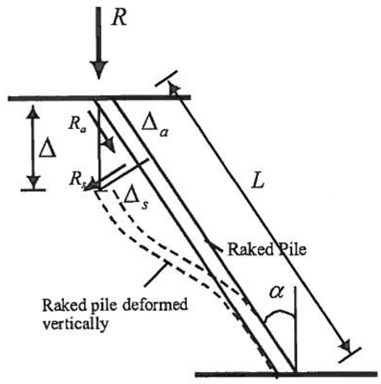

Fig. 2: Vertical deformations of a raked pile

3.2 Coefficient of uniform compression for a bearing end with moment fixity

The stiffness equation for axial deformation is given in equation 2

$$R_a = \left(\frac{EA}{L}\right)\Delta_a \tag{2}$$

Since,

$$\Delta_a = \Delta\cos\alpha$$

$$R_a = \left(\frac{EA}{L}\right)\Delta\cos\alpha \tag{3}$$

where E is Young's modulus of the pile, A is the cross-sectional area of the pile, L is the length of the pile and α is the angle of rake.
Similarly for shear, the stiffness is given by equation 4.

$$R_s = \left(\frac{12EI}{L^3}\right)\Delta_s \tag{4}$$

Since,

$$\Delta_s = \Delta\sin\alpha$$

$$R_s = \left(\frac{12EI}{L^3}\right)\Delta\sin\alpha \tag{5}$$

As shown in Fig. 2, R_a and R_s are components of R

$$R = R_a\cos\alpha + R_s\sin\alpha$$

Substituting equations 3 and 5,

$$R = \left(\frac{EA}{L}\right)\Delta\cos^2\alpha + \left(\frac{12EI}{L^3}\right)\Delta\sin^2\alpha$$

Rearranged,

$$R = \left[\left(\frac{EA}{L}\right)\cos^2\alpha + \left(\frac{12EI}{L^3}\right)\sin^2\alpha\right]\Delta = k_v\Delta \tag{6}$$

The vertical stiffness is the quantity in brackets. The total stiffness is the sum of the stiffnesses of each pile

$$k_v = \sum_1^n\left[\left(\frac{EA}{L}\right)\cos^2\alpha + \left(\frac{12EI}{L^3}\right)\sin^2\alpha\right] \tag{7}$$

where n is the number of piles. The pile group vertical stiffness is expressed in the same form as the subgrade reaction

$$c_u = \frac{k_v}{A_p} \tag{8}$$

where A_p is the cross-sectional area of the pile group.

3.3 Coefficient of uniform compression for a pinned bearing end

The derivation for the case of a pinned bearing end is similar to the fixed case. The difference is the term of the equation which represents the shear stiffness. The stiffness equation for a pinned bearing end is given in equation 9:

$$R = \left[\left(\frac{EA}{L}\right)\cos^2\alpha + \left(\frac{3EI}{L^3}\right)\sin^2\alpha\right]\Delta = k_v\Delta \tag{9}$$

The stiffness of the pile group is the sum of the stiffnesses of each pile.

$$k_v = \sum_1^n\left[\left(\frac{EA}{L}\right)\cos^2\alpha + \left(\frac{3EI}{L^3}\right)\sin^2\alpha\right] \tag{10}$$

Eq. 10 is converted into an equivalent subgrade reaction by equation 8.

3.4 Coefficient of non-uniform compression

The coefficient of non-uniform compression is determined for problems where a non-uniform pressure is applied to the soil. Rocking and pitching moments will cause non-uniform pressures. The foundation block spring system, subjected to a moment, is illustrated in Figure 3. As before, the springs represent the elastic stiffnesses of the piles.

3.5 Coefficient of non-uniform compression for fixed and pinned bearing end

If a rocking or pitching moment (*M*) is applied, the foundation block will undergo a rotation (ϕ). The moment will produce a vertical deflection Δ and a reaction *R* in each pile. The moment contribution of a single pile is given in equation 11.

$$M_i = Rl \tag{11}$$

where *l* is the distance from the moment axis to the location of the pile.
Since the pile will undergo a vertical deformation, the reactive force is equal to equation 6 or 9, depending on the end condition of the piles. Substituting equations 6 or 9 into 11,

$$M_i = k_v\Delta l$$

For small angles of rotation,

$$\Delta = \phi\, l \qquad M_i = k_v l^2\phi \tag{12}$$

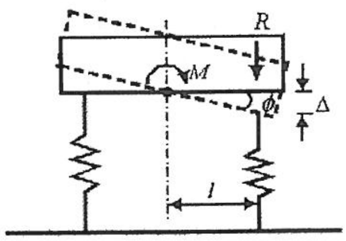

Fig. 3: Foundation spring system for rocking or pitching deformations

The rotational stiffness is therefore,

$$k_\phi = k_v l^2$$

The total moment (M) is equal to the sum of equation 12 for each pile.

$$M = \sum_1^n \left(k_v l^2\right) \phi \tag{13}$$

The rotation ϕ is a constant and therefore pulled out of the summation. From Barkan (Barkan DBF1962) [7], the moment is also expressed in terms of the subgrade reaction and the moment of inertia (I_p) of the pile group.

$$M = c_\phi I_p \phi \tag{14}$$

Equating equations 13 and 14, the coefficient of non-uniform compression is solved

$$c_\phi = \frac{\sum_1^n \left(k_v l^2\right)}{I_p} \tag{15}$$

The values l and I_p may change in the x and y directions. For this reason, equations 16 and 17 are defined accordingly

$$c_{\phi x} = \frac{\sum_1^n \left(k_v l_y^2\right)}{I_{px}} \tag{16}$$

$$c_{\phi y} = \frac{\sum_1^n \left(k_v l_x^2\right)}{I_{py}} \tag{17}$$

From equation 14, the rotational stiffness is determined.

$$k_\phi = c_\phi I_p \tag{18}$$

Substituting equations 16 and 17 into 18, the x and y stiffness' for non-uniform compression is solved

$$k_{\phi x} = \sum_1^n \left(k_v l_y^2\right) \tag{19}$$

3.6 Coefficient of uniform shear

The coefficient of uniform shear is derived from the interaction of the horizontal forces on the soil. The foundation block spring system is illustrated in Figure 4. As shown, the springs are placed horizontally to represent the shear stiffness of the soil. A raked pile is assumed to be deformed in the horizontal direction as shown in Figure 5. Similar to the case of uniform compression, the total deformation is broken down into two fundamental deformations—axial and shear (side sway). The axial deformation is represented by the symbol Δ_a and the shear deformation is represented by Δ_s. These deformations are associated with reactions R_a (axial) and R_s (shear).

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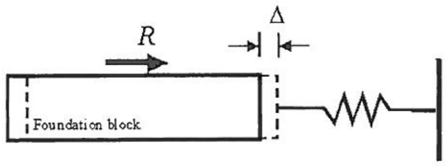

Fig. 4: Foundation spring system for horizontal deformations

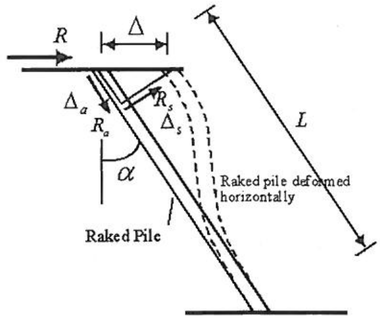

Fig. 5. Horizontal deformations of a raked pile

3.7 Coefficient of uniform shear for fixed bearing end

The stiffness equation for axial deformation is given by equation 20

$$R_a = \left(\frac{AE}{L}\right)\Delta_a \tag{20}$$

Since,

$$\Delta_a = \Delta \sin\alpha \qquad R_a = \left(\frac{AE}{L}\right)\Delta \sin\alpha \tag{21}$$

The stiffness for the shear (side sway) is given by equation 22

$$R_s = \left(\frac{12EI}{L^3}\right)\Delta_s \tag{22}$$

Since,

$$\Delta_s = \Delta \cos\alpha \qquad R_s = \left(\frac{12EI}{L^3}\right)\Delta \cos\alpha \tag{23}$$

The reactions R_a and R_s are components of R

$$R = R_a \sin\alpha + R_s \cos\alpha$$

Substituting equations 21 and 23

$$R = \left(\frac{AE}{L}\right)\Delta \sin^2\alpha + \left(\frac{12EI}{L^3}\right)\Delta \cos^2\alpha \tag{24}$$

Rearranged,

$$R = \left[\left(\frac{AE}{L} \right) \sin^2 \alpha + \left(\frac{12EI}{L^3} \right) \cos^2 \alpha \right] \Delta = k_h \Delta \tag{25}$$

The horizontal stiffness (k_h) is the quantity in brackets. The total stiffness is the sum of the stiffnesses of each pile

$$k_h = \sum_{1}^{n} \left[\left(\frac{AE}{L} \right) \sin^2 \alpha + \left(\frac{12EI}{L^3} \right) \cos^2 \alpha \right] \tag{26}$$

The horizontal stiffness of the pile group is expressed in the same form as the subgrade reaction

$$c_r = \frac{k_h}{A_p} \tag{27}$$

If the piles are raked, the horizontal shear stiffness may differ in the x and y directions, depending on the direction of rake. For this reason, the horizontal stiffnesses are determined in the x and y directions (k_{hx} and k_{hy}).

In the direction of rake, the pile stiffness is based on the axial and side sway deformations of the pile. If the pile is not raked, the stiffness equation will only include side sway. To account for this, the rake angle α is replaced by $\alpha \cos^2 \beta$ to determine the horizontal stiffness in the x direction and α is replaced by $\alpha \sin^2 \beta$ to determine the horizontal stiffness in the y direction. The angle β is the angle in plan of a raked pile, as shown in Figure 6. The horizontal stiffnesses in the x and y directions are given in equations 28 and 29.

$$k_{hx} = \sum_{1}^{n} \left[\left(\frac{AE}{L} \right) \sin^2(\alpha \cos^2 \beta) + \left(\frac{12EI}{L^3} \right) \cos^2(\alpha \cos^2 \beta) \right] \tag{28}$$

$$k_{hy} = \sum_{1}^{n} \left[\left(\frac{AE}{L} \right) \sin^2(\alpha \sin^2 \beta) + \left(\frac{12EI}{L^3} \right) \cos^2(\alpha \sin^2 \beta) \right] \tag{29}$$

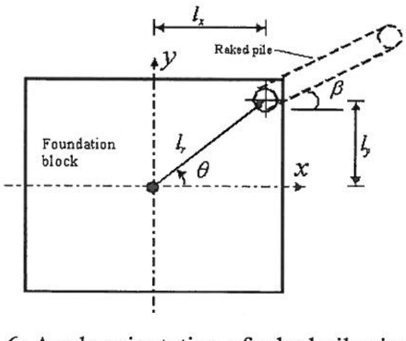

Fig. 6: Angle orientation of raked piles in plan

Furthermore, the coefficients of subgrade reactions are defined in the x and y directions

$$c_{rx} = \frac{k_{hx}}{A_p} \tag{30}$$

$$c_{ry} = \frac{k_{hy}}{A_p} \tag{31}$$

3.8 Coefficient of uniform shear for pinned bearing end

The derivation of the coefficient of uniform shear for a pinned bearing end is similar to the derivation of the fixed bearing end case. The difference is the horizontal shear stiffness term

$$k_{hx} = \sum_{1}^{n}\left[\left(\frac{AE}{L}\right)\sin^2(\alpha\cos^2\beta) + \left(\frac{3EI}{L^3}\right)\cos^2(\alpha\cos^2\beta)\right] \tag{32}$$

$$k_{hy} = \sum_{1}^{n}\left[\left(\frac{AE}{L}\right)\sin^2(\alpha\sin^2\beta) + \left(\frac{3EI}{L^3}\right)\cos^2(\alpha\sin^2\beta)\right] \tag{33}$$

The equivalent coefficient of subgrade reaction is determined by substituting equations 32 and 33 into equations 30 and 31.

3.9 Coefficient of non-uniform shear for fixed and pinned bearing end

The coefficient of non-uniform shear represents the stiffness of the soil determined from the twisting (or drilling or yawing) action of the foundation base. The applied moment is referred to as the yawing moment. The foundation block spring system is illustrated in Figure 7. From the figure, the radius (l_r) is the distance from the axis of the yawing moment to the location of the pile. The direction of deformation (Δ) and the calculated stiffness is orientated at right angles to the radius. The deformation is a function of the shear reaction (R_s) and the angle of twist (ψ).

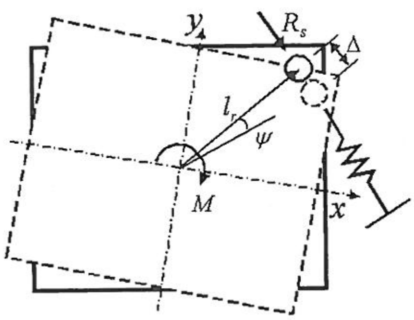

Fig. 7: Foundation spring system of twisting deformations

The moment contribution of a single pile is equal to the horizontal shear reaction times the radial moment arm (l_r).

$$M_i = R_s l_r$$

The horizontal reaction is equal to equation 25, at right angles to the radius

$$M_i = k_{h\theta} \Delta l_r$$

Where $k_{h\theta}$ is the horizontal stiffness at an angle θ (at right angles to the radius).
For a small angle of twist,

$$\Delta = l_r \psi \qquad M_i = l_r^2 \; k_{h\theta} \; \psi \tag{34}$$

The total yawing moment is equal to the sum of M_i for each pile.

$$M = \sum_1^n \left(l^2 k_{h\theta} \right) \psi \tag{35}$$

From Barkan[7], the twisting moment is related to the polar moment of inertia (J) and the subgrade reaction c_s.

$$M = c_s J \psi \tag{36}$$

Where J is the polar moment of inertia of the pile group. Equating equations 36 and 37, the coefficient of non-uniform shear is determined.

$$c_s = \frac{\sum_1^n \left(l_r^2 k_{h\theta} \right)}{J} \tag{37}$$

From equation 37, the twisting stiffness is determined

$$k_\psi = c_s J \tag{38}$$

Combining equations 38 and 39,

$$k_\psi = \sum_1^n \left(l_r^2 k_{h\theta} \right) \tag{39}$$

The horizontal stiffness ($k_{h\theta}$) is the stiffness at right angles to the radius (l_r)

$$k_{h\theta} = k_{hx} \sin^2 \theta + k_{hy} \cos^2 \theta \tag{40}$$

4 VALIDATION OF STIFFNESS EQUATIONS

A simple pile configuration, supporting a vibrating foundation, is illustrated in Fig. 8. The four piles are raked (1:5), the effective length 2.65 metres long (2.7 m along the rake) and assumed fixed at both ends. The pile is end bearing, cast into an underlying rock stratum and assumed fully fixed at the pile tip. The piles are raked in one direction along the x-axis. Other parameters, defining the properties of the piles are given below:

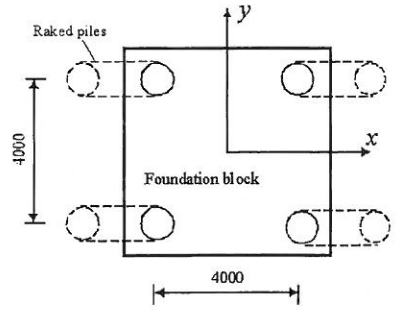

Fig. 8: Pile configuration of a foundation block

Pile diameter = 600 mm
f_{cu} = 30 MPa
E (dynamic) = 38 GPa
$I = 6.36 \times 10^{-3}$ m^4 (single pile)
$I_{px} = I_{py} = 4.55$ m^4 (pile group)
$A = 2.83 \times 10^{-1}$ m^2 (single pile)
$A_p = 1.13$ m^2 (pile group)
$J = 1.27 \times 10^{-2}$ m^3 (pile group)

Table 1: Comparison of stiffness equations

Pile Stiffness (4 pile grouping)	Stiffness equations (i.e., Eqns 7, 16, 17, 26, 27 and 36) (N/m)	Finite Element Solution (N/m)
k_v	1.53×10^{10}	1.54×10^{10}
$k_{\phi x}$	6.12×10^{10}	6.32×10^{10}
$k_{\phi y}$	6.12×10^{10}	6.32×10^{10}
k_{hx}	1.18×10^{9}	1.18×10^{9}
k_{hy}	5.89×10^{8}	5.88×10^{8}
k_{ψ}	7.07×10^{9}	7.05×10^{12}

The proposed stiffness equations are compared to a finite element solution. The model uses beam elements to represent the piles and shell elements (with an exaggerated E to make the block rigid) to model the block foundation. The stiffnesses of the pile group are determined by applying arbitrary forces to the finite element configuration and solving for the deformations (or rotations). The stiffnesses are calculated by dividing the force by the deformation or by dividing the moment by the rotation. The results are compared with the proposed theory and compiled in

Table 1. Since the derivation of the theory is based on "first principles," the equations are nearly identical to the finite element model.

5 CONCLUSIONS

The proposed equations are a simplified method to model piling in a form that is adaptable to CP 2012. The theory, however, dictates that the pile must be end bearing, unrestrained along the shaft and fixed or pinned at the bearing end. Although many pile types rely on skin friction, piles subjected to vibrations potentially lose the frictional bond between the soil and the shaft. For this reason, an unrestrained shaft is assumed [1]. Table 1 indicates that the proposed equations (which are derived from first principles) compare well with a finite element solution and therefore gives credence to the theory.

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