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The use of stereophotogrammetry to measure movements

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Synopsis

A review is made of the use of stereophotogrammetry to measure the movements of civil engineering works. An improvement to the false parallax technique of stereophotogrammetry, and a method of using a type of stereoplotter common in South Africa, the Wild B8, for this technique, are presented.

Samevatting

'n Oorsig van die gebruik van stereofotogrammetrie om die bewegings van siviele ingenieursbouwerke te meet is gemaak. 'n Verbetering aan die valse parallakstegniek van stereofotogrammetrie, en 'n metode vir die gebruik van 'n stereostipmasjiene in algemene gebruik in Suid-Afrika, die Wild B8, vir hierdie tegniek word aangebied.

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Introduction

The evidence of failure of most civil engineering works is usually excess movement. Consequently the measurement of movements is very important to check that theoretical predictions of behaviour, both in laboratory modelling and full scale works, is correct. However such movements are usually very small compared to the overall size of the works. This, together with the complexity and inaccessibility of many civil engineering works, frequently makes movement monitoring by direct measurement techniques very difficult.

Stereophotogrammetry offers the following advantages: time spent on site actually taking the photographs is small, and movements of all parts visible in the photographs can be measured. The location of those regions where movements have occurred may not be known at the time of visiting the site to take the photographs, but when the photographs are examined, a concentration of measurements can be made in the desired areas. Furthermore, a permanent record of the state of the works at a particular time is kept on the photographs, so that retrospective movement measurements can be made if they are required.

Review of Stereophotogrammetric techniques to measure movement

Two basic techniques are available. The first, illustrated in Fig 1, is to use successive stereo pairs of photographs of the object. A pair may be taken using two cameras simultaneously, or, if the time for the movement to occur is long, one camera may be used for both photographs. The positions in three dimensions, of salient features of the object, are measured from a stereo pair by conventional stereophotogrammetric methods. After movement may be expected to have occurred, another pair of photographs is taken. The positions of the same salient features are measured again, and their movements relative to their earlier positions can then easily be found.

This technique suffers from the disadvantage that the movements need to be significantly large relative to the overall size of the object in the photographs. The range of accuracy of measurement of a point in space, using terrestrial stereophotogrammetry, has been given by Cheffins and Chisholm[1] as lying between 1/1 000 and 1/20 000 of the distance from camera to object. They define accuracy as the standard deviation of the error from the true value, and this definition is used throughout this paper. Variation within the range depends upon the degree of difficulty of the project, the amount of expertise brought to it, and the time (i.e. money) spent on it. They further state that for 'normal average working conditions', an accuracy fraction of 1/10 000 can easily be achieved, but that in their experience, the funds generally assigned place the accuracy fraction at about 1/5 000.

These accuracies are perhaps more easily appreciated when the fraction of the distance from the camera to the object is converted to a proportion of the width of the object viewed by the camera. This can be done approximately by multiplying the fraction by the ratio of (focal length)/(format size) for the camera being used. This ratio can vary widely. From an examination of sixteen cameras used for terrestrial photogrammetry it was found to vary from 0.4 to 1.9, with a mean of 0.92. Hence for rough estimation purposes, the accuracy of normal terrestrial stereophotogrammetric measurement can be taken as 1/5 000 of the width of the object being photographed.

For the majority of structures, these figures mean that an unacceptably high number of photographs would be necessary to measure normal in-service structural movements. However, Moore[2], succesfully used this technique to monitor the movements of a 90 metre

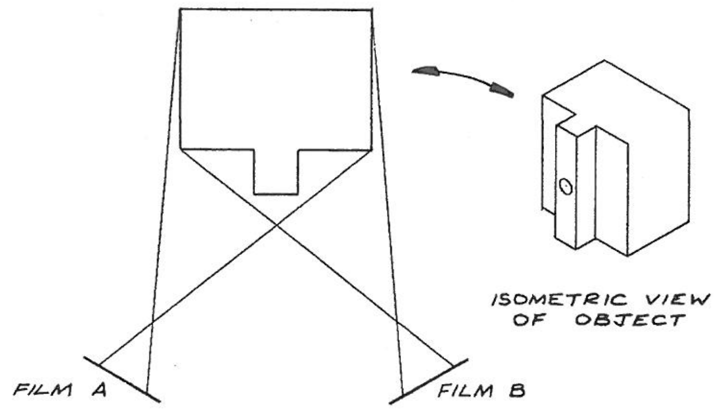

Fig 1(a): Plan of stereo pair production

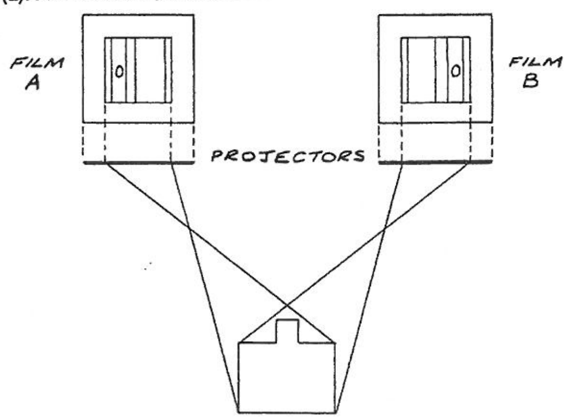

Fig 1(b): Projection of photographs to form stereo image

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high rockfill dam and its abutments during construction and filling. An accuracy of movement measurement of 50 mm was achieved, which was considered acceptable for the purpose. (Total movements were up to one metre.) Brandenberger[3] achieved an accuracy of 20 mm when monitoring the in-service behaviour of a similar dam with stereophotogrammetry, compared to an accuracy of 10 mm achieved on the same site with a theodolite and level survey.

The second technique uses single photographs of the object, taken at time intervals, from the same position, and in the same direction on each occasion. When two successive photographs are placed in an optical device that enables the viewer to see one photograph only with each eye, the relative movement of features between the successive photographs, in directions parallel to a line through the centres of the viewer's two eyes, has the same effect as parallax, and produces a stereo image. The depth of this stereo image can be measured by any of the instruments used for normal stereophotogrammetry, and the movement calculated from it. This technique has been called "false parallax", "motion parallax" or "time parallax".

The reason for the production of this stereo image can be appreciated by first considering the way in which a measurable stereo image is produced using the first (i.e. normal) terrestrial stereophotogrammetric technique. Fig 1a illustrates the production of a stereo pair of photographs of a three dimensional object, and Fig 1b shows the reconstituion of a stereo image using the photographs and two projectors.

The false parallax method is illustrated in Fig 2, where diagram 2a shows the production of a 'time-lapse' pair of photographs. The solid lines represent the initial position of the moving object, as seen on photograph A, while the dotted lines represent the final position of the moving object, as seen on photograph B. Fig 2b then shows these photographs being superimposed by two projectors. The moving object appears to be above the background that remains stationary. If there were a continuous pattern of movement points, then a continuous three dimensional image surface would be formed, such as that shown in Fig 6.

If the movement had occurred in the *y* direction as well as the *x* direction, (see Fig 3a), then when the photographs were placed in the projector as shown in Fig 3b, only the *x* direction movement would produce a stereo image, because a line through the eyes of the person viewing the stereo image, i.e. parallel to the stereo instrument's *X* axis would be parallel to the *x* axis on the photographs. To measure the *y* direction movement, the photographs must be rotated so that the *y* axis of the photographs is parallel to the eye base of the viewer, as shown in Fig 3c.

The difference of height in the stereo image, between a salient point of interest on the moving body and a point that is known to have been stationary, is directly proportional to the movement. The actual value of the movement can be obtained in two ways. The first way requires that the movement of a salient point on the moving body must be measured by independent means. Then, when measuring the ensuing false parallax stereo image, the ratio of that movement to the difference in level between the salient point and a stationary point defines the scale for the rest of the stereo image.

The second way of obtaining the movement can be applied when using a stereo image measuring instrument that measures the level of the image with a device called the 'floating mark'. This is the case for all modern instruments, and in fact, for the simplest instrument of all, the

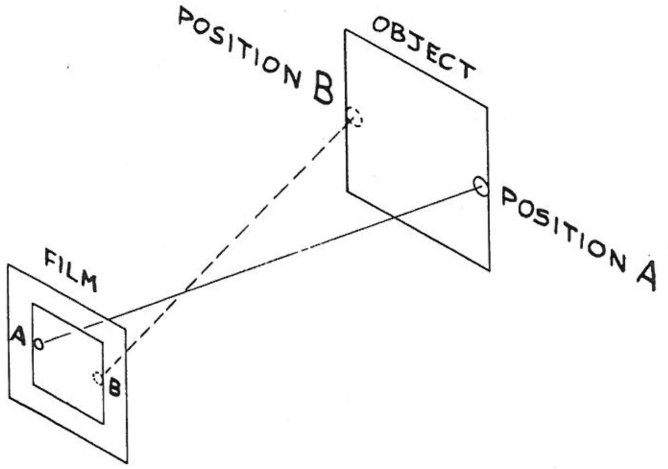

Fig 2(a): Isometric view of producing a time-lapse pair

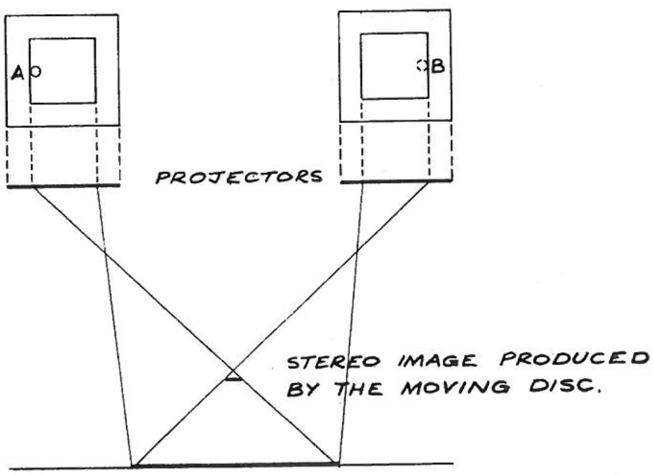

Fig 2(b): Projection of photographs to form stereo image

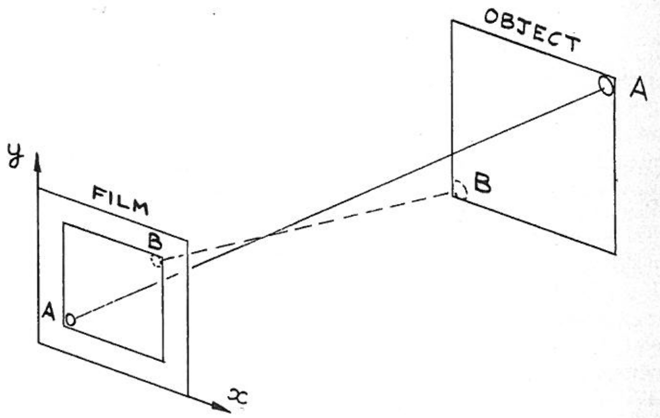

Fig 3(a): Time-lapse pair production

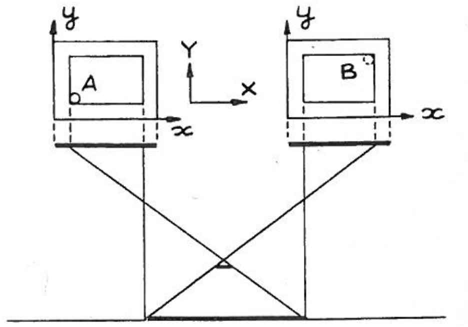

Fig 3(b): *x* movement measurement

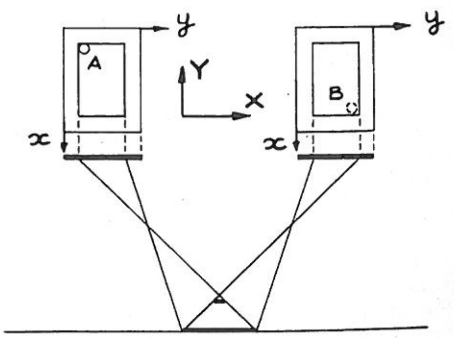

Fig 3(c): *y* movement measurement

parallax bar. In these instruments, the photographs are viewed through two optical trains that enable each eye to look at each photograph separately. This system is illustrated in Fig 4, where the 'cameras' are the beginning of the optical trains bringing the images of the photographs to the eyes. Within each optical train, a black dot is marked on one of the lenses. When both eyes, via the optical trains, are looking at exactly the same point on both photographs, then the black dots will be combined into one, and it will appear to be at the same elevation as the surface of the stereo image. By moving the camera ends of the optical trains slightly apart, the black dot will appear to sink in the stereo image, and by bringing them together, it will appear to rise. Hence the name floating mark is given to it. The perception of when the floating mark is coincident with the surface of the stereo image is a physical skill that varies with the individual and the amount of training and practice he has had.

Now the change in the distance between the instrument's cameras when the floating mark is moved in elevation from, say, a stationary point in the stereo image to a salient point, is the actual movement of that salient point at the scale of the photograph. That is, if the photograph is one tenth full size, then the actual movement of the object is ten times the change in distance between the cameras. Sometimes this change in distance can be measured directly on the instrument, (e.g. the parallax bar and all types of stereo comparaters). However, other types of instruments have been designed primarily for producing maps from aerial photographs, and this change in distance must be calculated from supplementary measurements on the instrument's geometry. (Eg the Wild A7 and B8 stereoplotters.)

Because its stereo image is produced by the relative movement only, the false parallax technique is inherently more accurate than the first mentioned. Also, because only one camera and one camera position are required, it is easier to carry out. Furthermore it can be done quite satisfactorily with inexpensive equipment. However, it suffers from the disadvantage that only movements in planes parallel to the plane of the camera's film can be measured. If any movements occur perpendicular to that plane they can cause significant errors. Nevertheless, this restriction still leaves a very wide range of applications.

The false parallax technique does not seem to have been frequently used, and there are few publications about it. In fact, two recent reviews[1,4] of methods of measurement, in fields where false parallax has been used successfully, made no mention of it. Also, Andrawes[5] implied that the method was an original idea of his and his co-workers in 1970[6], whereas a statement was made by Hallert[7] in 1954 that the method was already then well known.

However, despite the apparent lack of *frequency* of application, it has had a wide *range* of applications. These include:

1. The movement of the sand at all depths behind a model retaining wall[5].
2. Surface velocities of water[8].
3. Deflections of an 8 m long model of a bifurcated box girder bridge[9].
4. Velocities of traffic at an intersection[10].
5. Movements of rock slopes[11].

In applications 1 to 5, only movements in a single plane parallel to the plane of the film in the camera were measured. This is the usual case, because points on the object at different distances from the camera appear on the film at different scales. However, provided the principal distance of the camera, (centre of lens to film), and the perpendicular distance from the camera to the moving points on the object are known, then a scale correction to the measured movements can be made. To measure the movements of rock slopes[11], a stereo pair of photographs was taken, and then after the movement had occurred, another single photograph was taken from one of the camera locations from which the pair was taken. The stereo pair was used to measure, by normal stereophotogrammetric method, the distance from camera to salient features on the rock slope. The time-lapse pair of photographs was then used to measure movements.

Accuracy of the false parallax method of movement measurement

It is pertinent to first record those items which *do* affect the accuracy of normal stereophotogrammetry, but do *not* affect the false parallax method. These are:

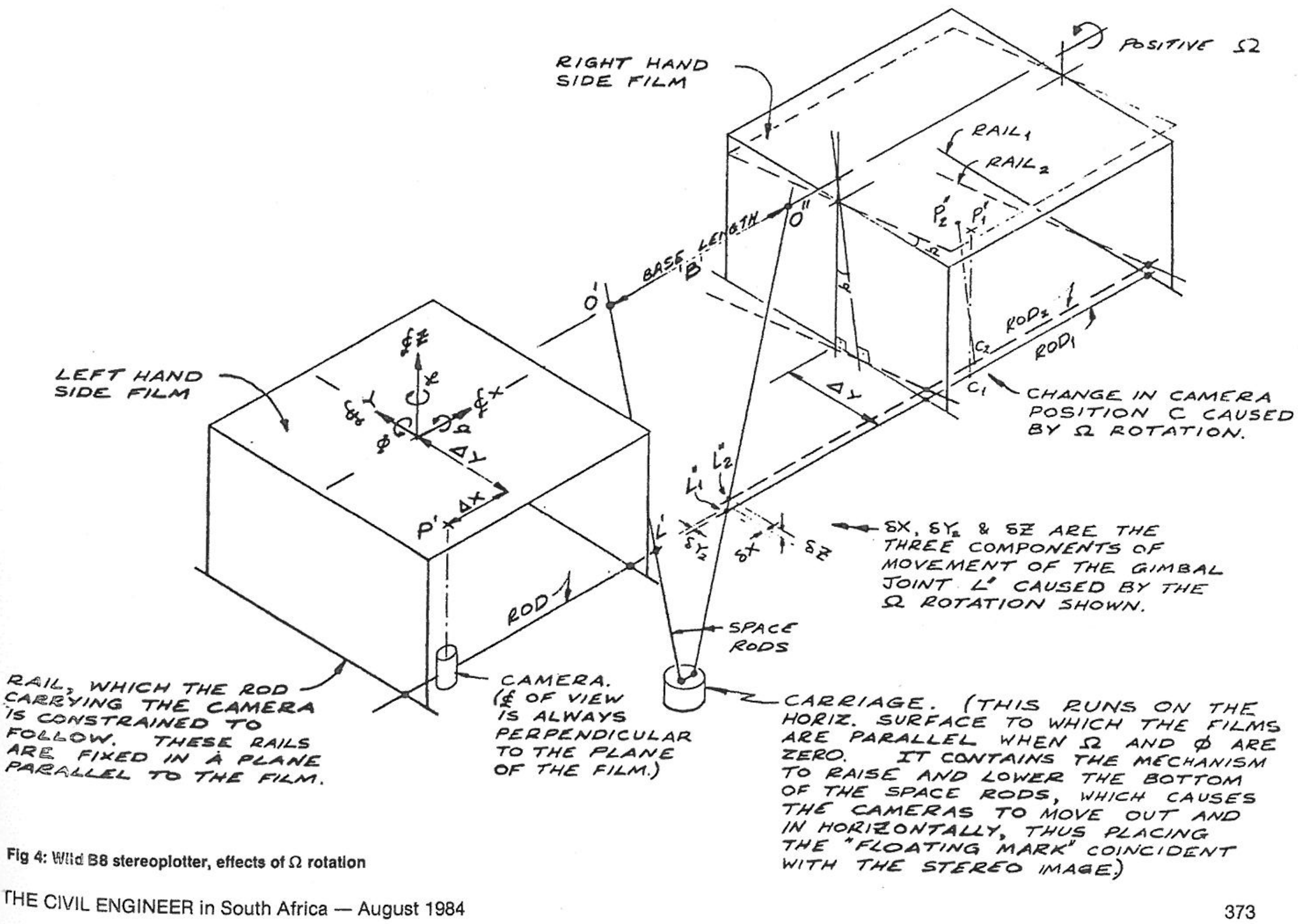

Fig 4: Wild B8 stereoplotter, effects of Ω rotation

1. The focal length of the camera does not need to be known.
2. The camera lens does not need to be distortion free, because the same part of the object is viewed through the same part of the lens in both of the time-lapse photographs. This greatly reduces the cost of the camera.
3. The distance from camera to object does not need to be known, provided all points move in the same plane parallel to the camera's film.
4. No grid of points of known positions needs to be in the photographs, provided the camera does not move between photographs.

Factors that do affect the accuracy of the false parallax method are as follows:

1. The tilt of the camera relative to the plane of the displacement field. A tilt of two degrees will produce an average error of approximately one per cent in the measured movements because of the scale variation already mentioned.
2. The film must be held flat when photographs are taken, and it must be stable, (i.e. its linear strain must be constant in all directions), when it is developed, and under temperature changes. Glass plates are ideal but good quality film (e.g. that used for aerial photography) is sufficient for most purposes.
3. If the camera must be moved between photographs, and cannot be reset exactly as before, then a set of points whose relative positions are known must appear in each time-lapse pair of photographs. This enables corrections to the measured movements to be made[9], but it does introduce a further possibility of error.
4. Operator skill in measuring the depth of the stereo image that represents the movement.

The usually attained accuracy of the false parallax method is better than that of normal stereophotogrammetry, because there are fewer potential sources of error, and it is not necessary to take the difference between measurements on two independent stereo pairs. In the case of false parallax, the accuracy is better expressed in terms of the relative movement on the film. Published case histories and my own experience show that an accuracy of 0.02 mm of relative movement on the film can easily be achieved with an ordinary camera and a good quality stereoplotter with an inexperienced operator. Accuracies of better than 0.010 mm can be achieved by an operator skilled in the measurement of stereo images, or by an inexperienced operator using the technique described in the next section.

A large film format size of 230 mm is commonly available, but will require an extension to the size of most cameras. Such an extension is not difficult to make for a camera that already has provision for either individually cut film or plates. Using this size of film, and with an accuracy on the film of 0.010 mm, the accuracy of movement measurement is approximately 1/15 000 of the size of the moving object, assuming that the object occupies 65 per cent of the film format.

The double grid technique for reduction of errors in the false parallax method

A grid of stationary objects is placed just in front of the moving object, so that the grid appears in both photographs of a time-lapse pair. When the pair is viewed stereoscopically, the grid appears as a horizontal plane in the stereo image, whereas any moving parts ideally appear as a 'valley' below the grid. (Refer to Fig 6). It is said 'ideally' because if the movement produces a valley stereo image rather than a hill, the image is easier to measure with the floating mark. The determination of whether a valley or a hill appears lies in the movement of the object on the photographs relative to the *X* axis of the stereo instrument. A hill can be changed to a valley simply by changing the sides of the stereo instrument that the photographs are placed on. The difference between the levels in the stereo image of a point on the 'valley bottom' and the grid above it, gives a very accurate measurement of the movement of that point.

The reason for the improvement in accuracy is that any distortions of the image on either of the two films will be common to both the stationary grid of objects and to the moving object. This applies to any form of distortion of the film emulsion, and to the effects of the film not being perfectly flat in the camera. Errors caused by these distortions have been found to be the most serious in terrestrial photogrammetry[12].

The fixed objects in front of the moving object must be sufficiently large so that the operator measuring their levels in the stereo image can

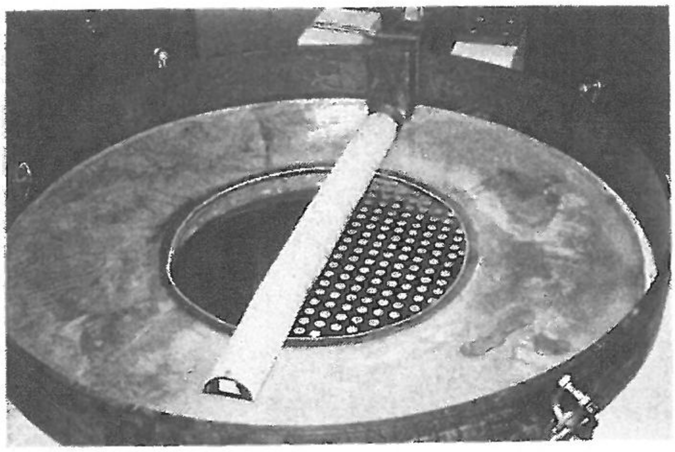

Fig 5: Apparatus to investigate the movements of sand round a loaded cylindrical anchor

do so without his vision being distracted by the adjacent stereo image of the moving objects. Conversely, they must not be so large that they obscure important details in the movement image. When using this technique to measure the movement of sand round a model cylindrical anchor behind a sheet of glass, and viewing the negatives at six times magnification, a grid of fixed objects 10 mm in diameter, and at 20 mm centres was found to be satisfactory. This experimental apparatus is shown in Fig 5.

Clearly the point of measurement of the height of the stationary grid image and the moving point's image cannot coincide because the former must obscure the latter. The problem is solved by interpolating between the heights of the stationary point images to give a stationary height value immediately over the image of the moving point. This interpolation can be done in a variety of ways. If the grid of stationary points is at regular centres, then linear interpolation between the heights of three stationary points can be used and is satisfactory for most purposes. A slight improvement on accuracy can be achieved by fitting a smoothing polynomial in one direction through the grid, and then, at points between the grid points, fitting a smoothing polynomial in the orthogonal direction. This method enables operator errors in measuring the stereo image to be smoothed out, and, if the measured heights are plotted, then gross errors (e.g. transposition of digits) can be spotted and individually corrected. If the grid of stationary points is not regular, then interpolation computer programs are available[13] which will accept random values of *x*, *y* and *z* coordinates and give values of *z* at any point defined by *x* and *y*. The simpler interpolation procedures mentioned can also be programmed onto a computer, and this saves much tedious labour.

Special correction needed for false parallax measurement with a stereoplotter common in South Africa

The fundamental principles of the false parallax method, together with the double grid technique, mean that virtually any camera can be used for the photographs. The sole remaining criteria are stability of camera between photographs, size of negative (the larger the better), and sharpness of image definition.

Similarly, any device for producing and measuring a stereo image, from the simple parallax bar to the electronically assisted stereo comparators, can be used to measure the stereo image produced by the false parallax. In the case of the stereo measuring instrument however, usually the more versatile and precise the instrument is, then the better will be the accuracy of the final results. One facility that is very important is to be able to remove *Y* false parallax. It is the relative movement of object points in the *X* direction (parallel to a line through the viewer's eyes), that produces the stereo image. However, in most cases, movements in the *Y* direction have also occurred. This *Y* false parallax causes the images of the two dots that make up the floating mark to diverge in the *Y* direction, even though the actual level of the floating mark may be at the level of the surface of the stereo image. The *Y* false parallax does not intrinsically cause an error. However it does make the measurement of the *X* false parallax, by determination of when the floating mark is on the surface of the stereo image, very difficult for the operator, which may cause inaccuracy to be introduced. *Y* false parallax can be removed by moving the time-lapse pair of photographs relative to each other in the Y direction while in the stereo measuring instrument.

A very common type of stereoplotter in South Africa is the Wild B8, which has all the facilities needed for X false parallax measurement, except the ability to easily remove the Y parallax. Now by rotating one photograph, (say the right hand side), relative to the other about the Ω axis in the stereoplotter, (see Fig 4), the point on the photograph viewed by the stereoplotter through its right camera will change in the Y direction. This will eliminate the Y false parallax. However this difference in Ω will cause a significant error unless a correction is made.

The Ω correction needed is derived from the geometry of the Wild B8 instrument in appendix A, and is given by:

$$\left[\begin{array}{l}\text{Stereo image height}\\ \text{for zero rotation}\end{array}\right] = \left[\begin{array}{l}\text{Measured stereo image}\\ \text{height for a rotation of } \Omega\end{array}\right] - C$$

where:

$$C = \delta x \; x \left[\frac{\begin{array}{l}\text{(No. of stereo-}\\ \text{plotter height}\\ \text{units represen-}\\ \text{ting a known}\\ \text{movement)}\end{array}}{\begin{array}{l}\text{(Value of the}\\ \text{known move-}\\ \text{ment)}\end{array}}\right] \times \left[\begin{array}{ll}\text{Photo. scale} & \text{(Length in}\\ \text{factor, i.e.} & \text{actual apparatus)}\\ & \overline{\text{(Length of film)}}\end{array}\right]$$

The correction parameter δx is the adjustment necessary to the distance between the instrument's cameras when using the second method of calculating the movements from the false parallax, described above, and is given by:

$$\delta x = \frac{\left(\frac{B}{2}\ \Delta X \frac{Z}{f}\right)}{Z} \left[-f \left| \frac{1-\cos\Omega}{\cos\Omega} \right| -\Delta Y \tan\Omega \right]$$

(The variables in this equation are shown in Figs 8 and 9.)

Example of the use of the double grid technique and the wild B8 stereoplotter

To investigate the movements of sand around a long cylindrical anchor, a model anchor was sectioned along its longitudinal centreline, and laid on a glass sheet as shown in Fig 5. A depth of sand of 500 mm was placed over the anchor, and it was then pulled axially so that it moved through the sand. Photographs were taken on 230 mm film, through the glass sheet, of the anchor and its surrounding sand, at intervals during the loading of the anchor. Paper discs, stuck on the outside of the glass, formed the stationary grid. The camera used had an original format size of 125 mm square. This was increased to 230 mm by means of a wooden extension box clamped to the back of the camera.

Fig 6 shows an insometric view of a typical three dimensional image that was produced when two successive photographs from these tests were viewed in the B8 stereoplotter. Note that the isometric view is used for clarity of illustration in this paper. When the image is viewed in the stereo instrument, the point of view is directly down onto the stereo image.

Measurements of the difference in stereo image height between the grid of fixed marks and the surface of the stereo image enabled the movements of the sand at distance from the anchor to be determined. From diagrams of these movements, information about the manner in which the soil is responding to stress can be obtained. For example, Fig 7a shows that the longitudinal soil movements caused by a cylindrical anchor are highly concentrated adjacent to the anchor, and so is the shear induced dilation that gives cylindrical anchors their remarkably high load carrying capacity.

The accuracy of the movement measurements was established by the following test. A piece of wooden chipboard was laid on the glass sheet, in place of the sectioned anchor, and moved a known amount in the x and y directions. A time lapse pair of photographs was taken of the wood and the stationary marks; one photograph before and one after the movement.

The two stereo images produced by the x and y movements of this time lapse pair were measured in the Wild B8 stereoplotter. The Ω rotation was used to remove the Y false parallax while measuring the X false parallax, and the correction described in appendix A was applied.

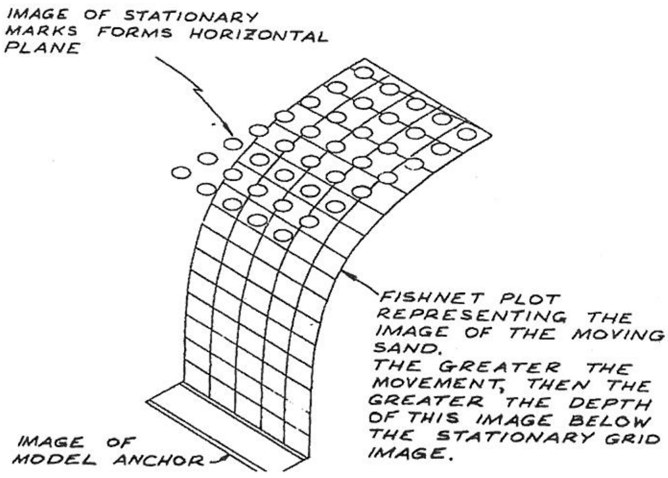

Fig 6: Isometric sketch of the stereo image produced by the false parallax caused by longitudinal movement of the anchor and surrounding sand

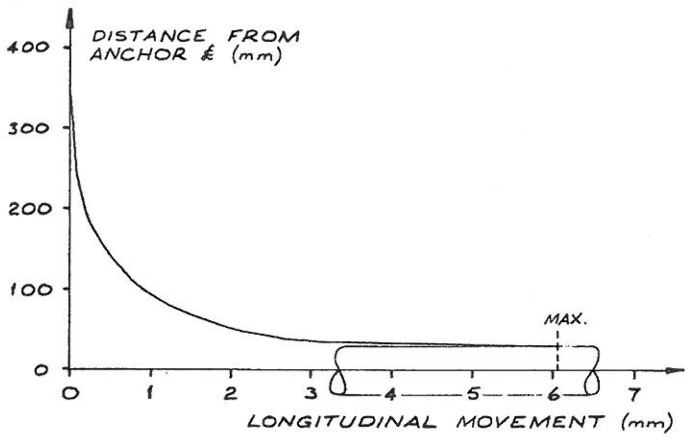

Fig 7(a): Total sand movement at peak shear stress

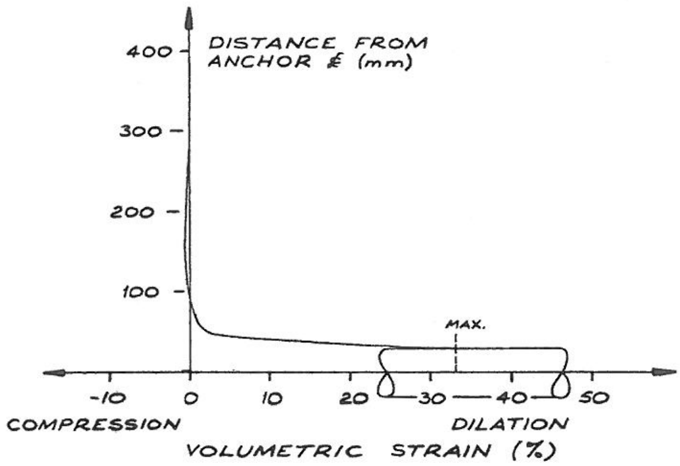

Fig 7(b): Total sand volumetric strain at peak shear stress

After the Ω correction had been applied, it was found that the standard deviation of the movement measurements, without using the double grid technique, was 0,02 mm. (That is, only using the height of the stereo image of the moving wood to establish the movement value.) However, by using the double grid technique, this error was reduced to 0,009 mm. (That is, by subtracting the height of the stereo image of the moving wood from the height of the stereoscopic image of the grid of stationary marks.)

By varnishing the wooden chipboard, a textured surface was obtained which gave a clear stereo image for measurement. When measuring the stereo image of moving sand it is not quite so easy, because the individual grains tend to roll unevenly, and dust in the sand makes the stereo image unclear. However, when measuring such images, the consistency of the values on remeasurement indicated that the accuracy of measurement was better than 0.015 mm. This accuracy was quite adequate for the purposes of the investigation, and in fact was better than the reproducibility of the tests.

Conclusions

Stereophotogrammetry is a very useful method of measuring the movements of civil engineering works. The false parallax technique of stereophotogrammetry, (using time lapse pairs of photographs), especially with a stationary grid in front of the moving object, has several advantages over the more normal technique of using stereo pairs of photographs. These are:

1. A simple and inexpensive camera can be used.
2. On-site photography is simpler.
3. It is inherently more accurate.

Although the false parallax technique ideally requires a *Y* shift facility in the stereo image measuring instrument, the Ω rotation can be used to effect a *Y* shift on a type of stereoplotter common in South Africa, providing a correction is applied to the measured movements.

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Appendix A

Correction for the Effect of Using Ω Adjustment to Remove *Y* Parallax between Photographs, while Measuring *x* Parallax with the Wild B8 Stereoplotter

Effect of Ω when cameras are on the x *centreline of the stereoplotter:* Fig 4, shows the effects of rotating the right hand film carrier by an angle Ω about the axis through 0'0", when the cameras are observing a point well away from the *x* and *y* centreline axes of the film carrier. Of the three possible rotations of the films, *x*, ∅ and Ω; *x* and ∅ are zero for both films, and only the right hand side film has experienced a rotation Ω. In this section, the simpler case of where the cameras initially look at the *x* centreline of the film carriers will be considered.

Fig 8 shows a view of the right hand side (R.H.S.) of the stereoplotter, but viewed from *its* left hand side. The solid lines represent the initial state, with the film completely parallel to the base of the stereoplotter, and the dotted lines represent the situation after a rotation of Ω has been made.

The reason for making the Ω adjustment, (in the present application of stereophotogrammetry), is to cause the R.H.S. camera to look at a point on the film a small distance away, in the *Y* direction, from where it was initially looking. In Fig 8, the camera at L_1 is initially looking at point A_1 on the film. After the Ω rotation, A_1 has moved to A_2 and the camera has moved to L_2 and is looking at point A_3.

Now, $A_3A_2 = L_2L_3$ = (focal length set on the stereoplotter) × $\tan\Omega$

i.e. Movement of point viewed on R.H.S.

$$= \delta Y_1 = f.\ \tan\Omega \qquad (1)$$

The camera moves vertically from L_1 to L_2, because it is constrained by the space rod. It is not important if the camera moves vertically, but in doing so, the cardan joint *L*, being constrained to follow the space rod, causes the camera to move in the *x* direction (refer to Fig 4). This *is* important because it changes the apparent height of the floating mark in the stereo image.

From Fig 8:

$$H_1 = L_2L_3 \sin\Omega$$
$$= f \tan\Omega \sin\Omega$$
& $$H_2 = f - f\cos\Omega$$
∴ $$H_1 - H_2 = f\left(\frac{\sin^2\Omega}{\cos\Omega} - 1 + \cos\Omega\right)$$

i.e. Vertical distance moved

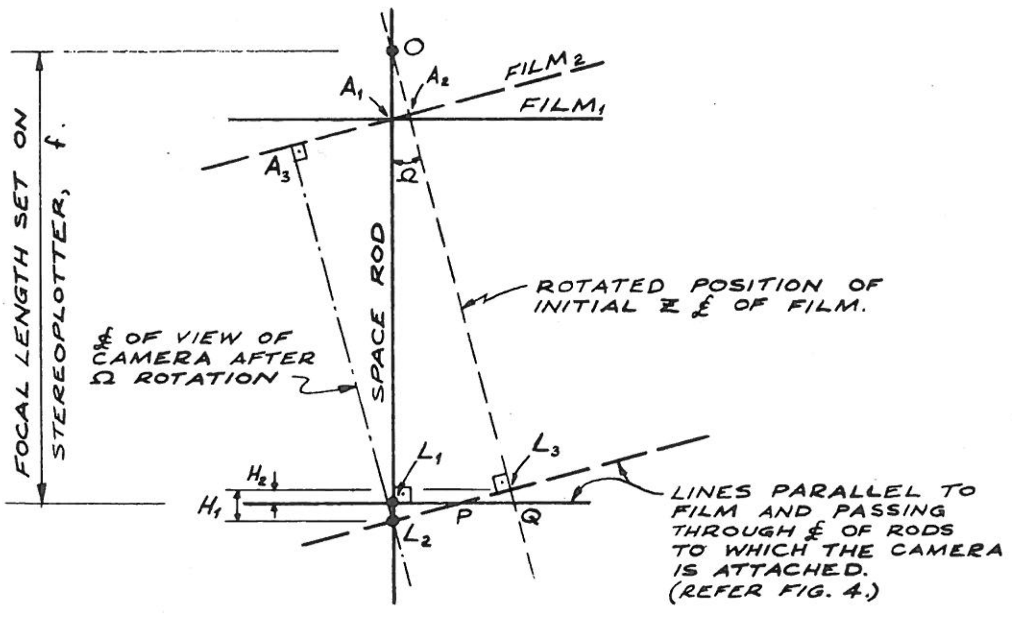

Fig 8: Schematic view of the right-hand side of the stereoplotter, seen from its left-hand side, showing the effect of an Ω rotation

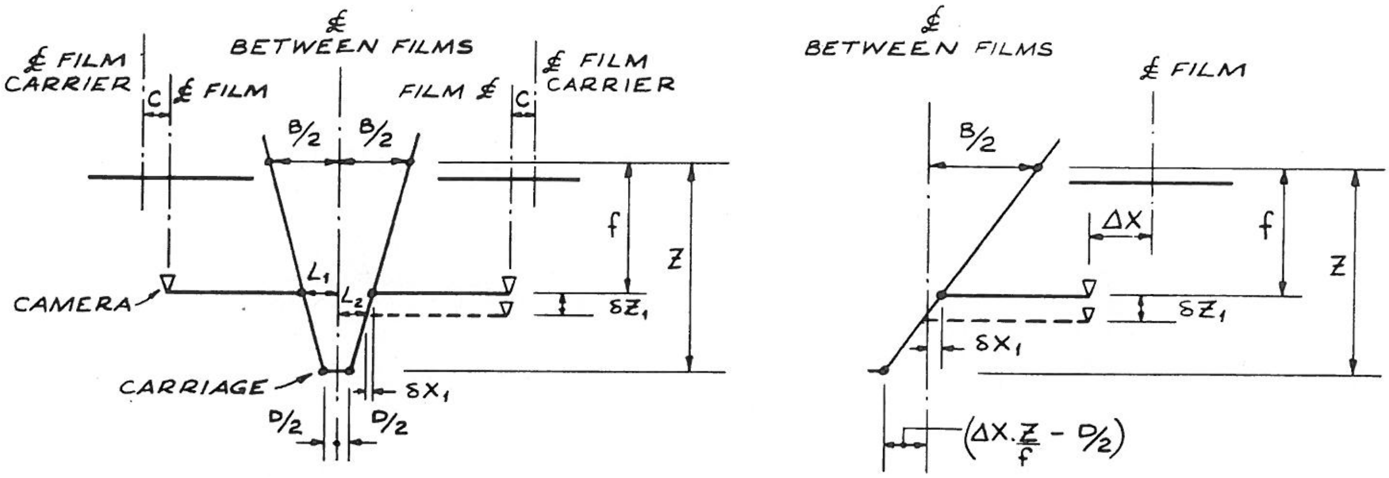

Fig 9(a): Carraige central between films

Fig 9(b): Carriage displaced along *X* centreline

Fig 9: Relationships between *X* and *Z* movements of the stereoplotter's camera

by camera, caused purely
by rotation of $\Omega = \delta Z$, $\quad = f \; \underline{1 - \cos\Omega}$ (2)

Now the movement of the camera in the *X* direction caused by this vertical movement will depend upon the slope of the space rod. Fig 9a shows a view from the front of the stereoplotter, when the carriage is central between the L.H.S. and R.H.S. films. Fig 9b shows the situation when the carriage is displaced a distance Δx along the *x* centreline, where Δx is measured from the *film* centreline.

from Fig 9b: $\quad \delta X_1 = \dfrac{\left(\frac{B}{2} - \frac{D}{2} + \Delta x \frac{Z}{f}\right)}{Z} \times \delta Z_1$

and using Eq 2: $\quad \delta X_1 = \dfrac{\left(\frac{B}{2} - \frac{D}{2} + \Delta x \frac{Z}{f}\right)}{Z} \times f \left(\dfrac{1 - \cos\Omega}{\cos\Omega}\right)$ (3)

Note that Fig 9a shows that when the carriage is on the centreline between *films*, the cameras are a distance *c* inside the actual centrelines of the film *carriers*. This is because the B8 stereoplotter is not designed to give 100 per cent coverage of the full width of the film carriers, since this is not required in most stereophotogrammetric applications. The optimum coverage is achieved when the base length, (*B* on Fig 3), is as *small* as possible, when a film width of ~175 mm in the *x* direction can be covered by the cameras. (This can be seen from Fig 4, if it is realised that points 0' and 0" are attached to the film carriers).

Effect of Ω when the cameras are at a distance ΔY from the x centreline of the stereoplotter: The two results described above: *Y* shift of viewed point and *X* shift of the camera, both remain constant if the carriage is now moved away from the *x* centreline of the stereoplotter.

However, as the rod carrying the camera travels along the inclined rail, (refer Fig 4), it moves vertically, thus moving the cardan joint *L"* from L''_1 to L''_2. Because the space rod is now inclined to the vertical plane through the *x* centreline of the stereoplotter, this movement causes a displacement of the camera in both the *x* and *y* directions.

Vert. movement of rod
carrying the camera $= \delta Z_2 = \Delta Y \tan\Omega$ (4)

Where ΔY is the distance from the *x* centreline of the stereoplotter of the point on the film seen by the camera if Ω were zero. In fact, because Ω is small, ΔY can be taken as the distance from the *x* centreline of the point seen by the camera *with* the Ω rotation having been made.

Then from Fig 9b:
x direction movement
of the camera $\quad = \delta X_2 = \dfrac{\left(\frac{B}{2} - \frac{D}{2} + \Delta x \frac{Z}{f}\right)}{Z} \times \delta Z_2$

and using Equ 4 $\quad = \delta X_2 = \dfrac{\left(\frac{B}{2} - \frac{D}{2} + \Delta x \frac{Z}{f}\right)}{Z} \times \Delta Y \tan\Omega$ (5)

The effects of a rotation Ω on the R.H.S. film carrier, in the general case, cannow be summarized, and written in terms of a consistent sign convention:

Change of position in *X* direction of point viewed on film =

$$\delta x = \delta x_1 + \delta x_2 = \frac{\left(\frac{B}{2} - \frac{D}{2} - \Delta x \frac{Z}{f}\right)}{Z} \left[-f \left| \frac{1 - \cos\Omega}{\cos\Omega} \right| - \Delta Y \tan\Omega \right] \quad (6)$$

Note:

1. δx is positive when the point viewed on the film moves in the positive direction of the *x* axis as shown on Fig 4.
2. Δx is positive when the camera is on the positive *x* side of the film centreline, as shown on Fig 4 and negative when the carriage is on the negative side of the film centrelines.
3. ΔY is positive when it is on the positive *Y* side of the *x* centreline axis.
4. The expression is written to give the correct sign for δx when values of Ω, as read from the scale on the B8 stereoplotter, are used. (The positive direction of Ω as given by the scale on the B8 is shown on Fig 4, on the L.H.S. film carrier).

Book review

***Precast concrete: Design and applications,* by A M Haas. First Edit vii + 152 pp with 23 tables and 412 illus. London, Applied Science Publishers 1983. No price given.**

In 13 chapters, which averages less than 12 pages per chapter, this book attempts to deal with a large number of aspects of prefabrication, including housing, bridges, caissons, shells, spatial structures, stability, modular co-ordination, tolerances, rationalization, standardization and mechanization. It is simply impossible to cover all these aspects and more in a slim volume such as this.

The book nevertheless makes interesting reading and a number of important principles are dealt with. The practising engineer with more than a superficial knowledge of precast concrete, however, will find very little new in the book. One may perhaps call it a "coffee table" book for browsing when you have nothing better to do. *F A Heymann*